Roller Coaster Physics
Lesson 8
Circular Loops

Let's look at a frictionless ***circular*** loop of radius *r* (Figure 1). We will assume further that the roller coaster train is traveling at a speed just fast enough so that the cars barely remain on the track at the top of the loop. If the train was going any faster, you would "feel" the seat and if it was going any slower, the car would fall off the track.

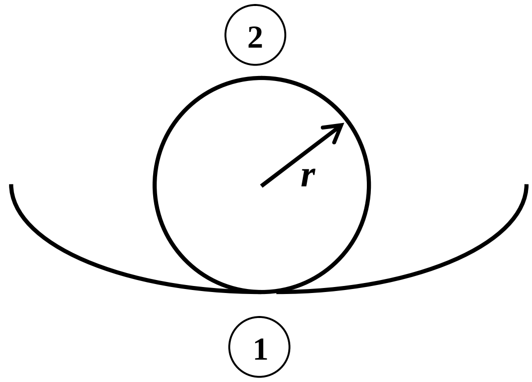

Figure 1

1. Let's look at what happens to the coaster train and passengers at the top of the loop, Point 2.
   a. At the top of the loop the centripetal forces working on the train are shown in Figure 2.

$$CF + W = F_c$$

   b. However, if the train is just barley remaining on the track, that means $CF = 0$. (Note: If $CF < 0$, then the train falls away from the track and if $CF > 0$, then it is pushed onto the track). Note also that if $CF = 0$, then $FF = 0$.

   We can now find an expression for the velocity at Point 2 (v_2) by evaluating the Centripetal Force on the coaster train when $CF = 0$.

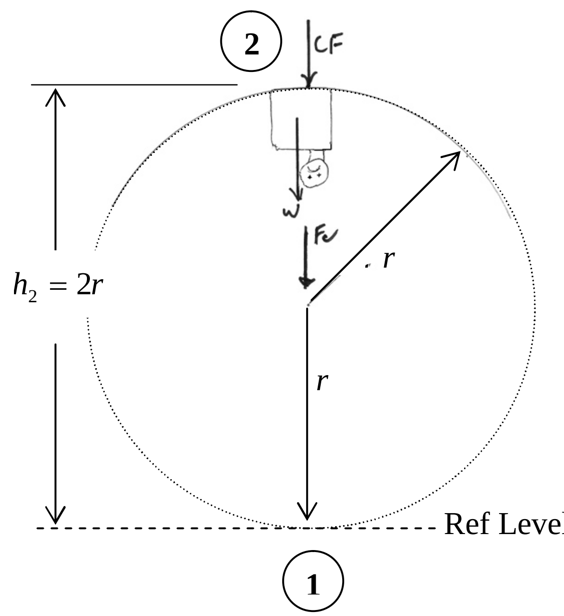

Figure 2

$$\cancel{CF}^{0} + W = F_C$$

$$W = F_C$$

But, $W = mg$ and

$$F_c = \frac{mv_2^{\,2}}{r}$$

$$\cancel{m}g = \frac{\cancel{m}v_2^{\,2}}{r}$$

$$v_2^{\,2} = gr$$

$$v_2 = \sqrt{gr} \quad \text{(for a } \textit{Force Factor} \text{ of 0)}$$

c. Next, we will find the kinetic and potential energies at Point 2.

$$KE_2 = \frac{1}{2}mv_2^{\,2}$$ But we found that $v_2^{\,2} = gr$. Thus

$$KE_2 = \frac{1}{2}mgr$$

And

$$PE_2 = mgh_2$$ But, from Fig. 2, we see that $h_2 = 2r$. Thus

$$PE_2 = mg(2r)$$

$$PE_2 = 2mgr$$

d. Now, let's look at the Total Energy at Point 2 (TE_2) which, again, is the total energy of the roller coaster train if it is moving slow enough that it is just on the verge of falling off the track:

$$\begin{aligned} TE_2 &= KE_2 + PE_2 \\ &= \frac{1}{2}mgr + 2mgr \\ &= \left(\frac{1}{2} + 2\right)mgr \\ TE_2 &= 2.5mgr \end{aligned}$$

2. Above we calculated the slowest speed the coaster train can go and still remain on the track. Let's examine what must happen to the coaster train and passengers at the bottom of the loop for this to happen.
   a. First, we will calculate what the velocity must be at the bottom of the circular loop, Point 1 (v_1), so that the car is just on the verge of falling off the track at the top of the loop, Point 2. We will write an Energy Balance equation between Points 1 and 2:

$$TE_1 + W_{1-2} = TE_2 \quad \text{but, } W_{1-2} = 0\text{, so}$$

$$TE_1 = TE_2 \quad \text{and } TE_1 = KE_1 + PE_1\text{, so}$$

$$KE_1 + PE_1 = TE_2 \quad \text{but, } PE_1 = 0\text{, so}$$

$$KE_1 = TE_2 \quad \text{but, from the previous page we know } TE_2 = 2.5mgr\text{, so}$$

$$KE_1 = 2.5mgr \quad \text{but, } KE_1 = \frac{1}{2}mv_1^2\text{, so}$$

$$\frac{1}{2}\cancel{m}v_1^2 = 2.5\cancel{m}rg$$

$$v_1^2 = 5rg$$

$$v_1 = \sqrt{5rg}$$

NOTE: This is the ***slowest*** speed the coaster can be going at the *bottom* of the loop to insure that it is just barely remaining on the track at the *top* of the loop. If v_1 were any slower, the speed at the top of the loop (v_2) would be to slow and the train would fall off the track.

   b. Next, let's look at the Force Factor at the bottom of the loop, Point 1, FF_1 (Remember, the Force Factor at the top of the loop is 0 in this case):

In the previous Lesson , we proved that

$$FF_1 = \frac{v_1^2}{rg} + 1$$

But for this case, where the train is going as slow as possible but still remains on the track at the top of the loop, we showed that $v_1^2 = 5rg$. So:

$$FF_1 = \frac{5\cancel{rg}}{\cancel{rg}} + 1$$

$$= 5 + 1$$

$$FF_1 = 6\ g\text{'s}$$

Oh Oh!!!!

3. **<u>CONCLUSION</u>**: If we pull 0 g's at the top of the circular loop (Point 2) then we must pull 6 g's at the beginning of the circular loop (Point 1). This is way to many g's and most of the passengers would pass out.

   If we slow the train down at the bottom of the loop (Point 1) to a reasonable Force Factor of, say 4 g's, the train will be going to slow at the top of the loop and will fall off the track.

   Therefore, ***circular*** loops are ***not possible*** and cannot used in roller coasters.