Lesson 7
Class Notes
Vertical Forces and Centripetal Acceleration

In amusement park rides like roller coasters, forces on your body are usually discussed in terms of a Force Factor.

$$\text{Force Factor }(FF) = \frac{\text{force the rider feels}}{\text{weight of the rider}}$$

1. If you are not accelerating, the ground or the chair you are sitting in must exert an upward force equal to your weight to keep you from falling. Thus,

   $F_{feel} = W$ so,

   $$\text{Force Factor} = \frac{F_{feel}}{W} = 1$$

   A force factor of 1 is normal. (Note the similarity to g-force)

2. If you are standing in an elevator that is accelerating upward, the floor of the elevator must push up on the feet with a fore greater than your weight and the force you feel is more than normal. In fact, if you were standing on a bathroom scale, it would read more than your normal weight. For example, if you "weighed" twice as much as normal, the force you feel is twice your weight or $F_{feel} = 2W$. Thus Force Factor $= \frac{F_{feel}}{W} = \frac{2W}{W} = 2$.

   Similarly, in order for a rider to accelerate upward, such as the bottom of a coaster hill or at the bottom of the swing of Da Vinci's Cradle, the chair must push up on you with a force that is greater than your own weight.

3. As you go over the top of a roller coaster hill, the force you feel will be less than your normal weight. For example, if you felt you weighed half of your normal weight then your Force Factor is ½:

   $$FF = \frac{F_{feel}}{W} = \frac{0.5W}{W} = 0.5$$

   This would indicate that the seat is pushing up with a force that is half the normal weight of the passenger and they will feel light.

Astronauts and test pilots talk about "pulling g's." Pulling 3 g's means the same thing as a force factor of 3. When discussing forces, force factor and "g's" are often used interchangeably.

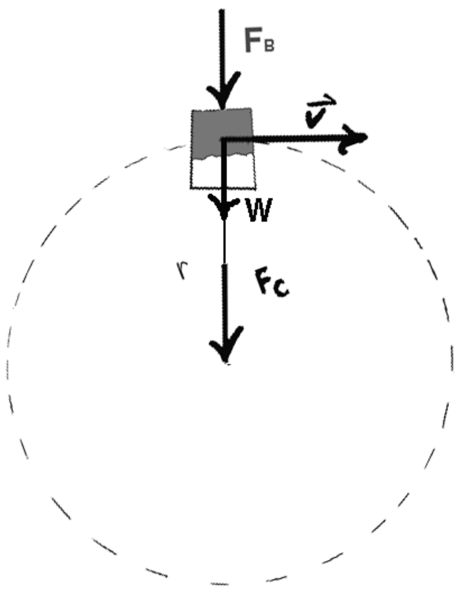
In this class we will refer to the force we feel as the ___Chair Force:___ ($CF = F_{feel}$).
So the Force Factor is found by:

$$\text{Force Factor }(FF) = \frac{\text{force the rider feels}}{\text{weight of the rider}} = \frac{F_{feel}}{W}$$

$$FF = \frac{CF}{W}$$

Centripetal Acceleration

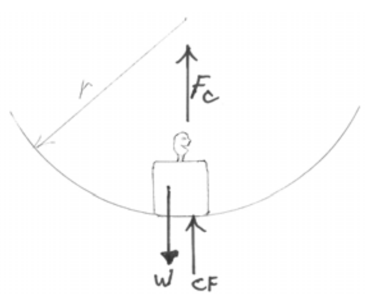
1. Consider a bucket containing a mass of water being swung in a vertical circle. The water does not fall out. The water tries to go in a straight line with a constant velocity (same speed and direction), $\vec{v}$. The bucket, however, keeps applying a force F_B toward the middle of the circle that makes the water change its velocity by changing its direction. This change in velocity is called C*entripetal Acceleration*. The force required to create this acceleration is called the *Centripetal Force.*



It can be shown that the magnitude of the centripetal acceleration of an object traveling at velocity v and in a circle of radius r is given by $a_c = \frac{v^2}{r}$. The direction of the acceleration is towards the center of the circle. According to Newton's Second Law $(F = ma)$, the centripetal force required to create this acceleration is given by: $F_c = m\frac{v^2}{r}$. This force also is always towards the center of the circle.

2. Now, let's find the forces acting on your body (in terms of Force Factor) sitting in a roller coaster car that is changing direction (accelerating) at the bottom of a hill.

From the drawing, we see that $CF - W = F_C$. Solving for CF : $CF = F_C + W$



By definition, $FF = \frac{CF}{W}$

So, if we substitute $F_c + W$ in for CF, we get

$$\frac{F_c + W}{W} = \frac{F_c}{W} + \frac{W}{W}$$

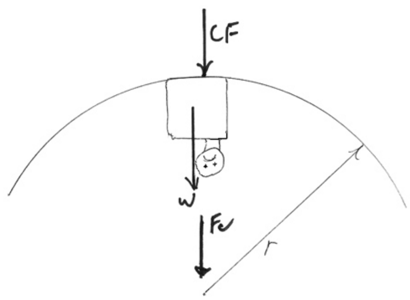
$$\frac{F_c}{W} + 1$$

but $F_c = \frac{mv^2}{r}$ and $W = mg$. So, if we substitute these expressions into the equation above, we will get:

$$FF = \frac{\frac{mv^2}{r}}{mg} + 1 = \frac{mv^2}{rmg} + 1$$

$$\boxed{FF = \frac{v^2}{rg} + 1}$$

3. Now, let's look at a roller coaster car at the top of a loop-to-loop and hanging upside down.



In this case: $CF + W = F_c$

Solving for CF :

$$CF + W - W = F_c - W$$
$$CF = F_c - W$$

Find FF:

$FF = \frac{CF}{W}$. So

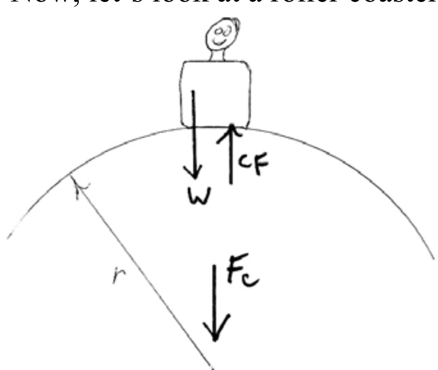
$$FF = \frac{F_c - W}{W} = \frac{F_c}{W} - \frac{W}{W} = \frac{F_c}{W} - 1$$

But, $F_c = \frac{mv^2}{r}$ and $W = mg$

$$FF = \frac{\frac{mv^2}{r}}{mg} - 1 = \frac{mv^2}{rmg} - 1$$

$$\boxed{FF = \frac{v^2}{rg} - 1}$$

4. Now, let's look at a roller coaster car at the top of a hill and right-side-up.



In this case: $W - CF = F_c$

Solving for CF :

$$W - CF + CF = F_c + CF$$
$$CF + F_c = W$$
$$CF + F_c - F_c = W - F_c$$
$$CF = W - F_c$$

Find FF:

$FF = \frac{CF}{W}$. So

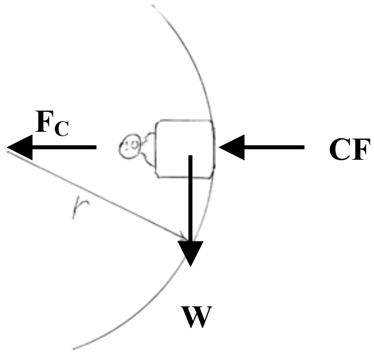
$$FF = \frac{W - F_c}{W} = 1 - \frac{F_c}{W}$$

But, $F_c = \frac{mv^2}{r}$ and $W = mg$

$$= 1 - \frac{\frac{mv^2}{r}}{mg} = 1 - \frac{mv^2}{mrg}$$

$$\boxed{FF = 1 - \frac{v^2}{rg}}$$

5. Now, let's look at a roller coaster car half way up a loop-to-loop.



In this case: W does not contribute to F_c.

Solving for CF :

$$CF = F_c$$

Find FF:

$$FF = \frac{CF}{W}. \quad \text{So}$$

$$FF = \frac{F_c}{W}$$

But, $F_c = \frac{mv^2}{r}$ and $W = mg$

$$FF = \frac{\frac{mv^2}{r}}{mg} = \frac{\cancel{m}v^2}{\cancel{m}rg}$$

$$\boxed{FF = \frac{v^2}{rg}}$$