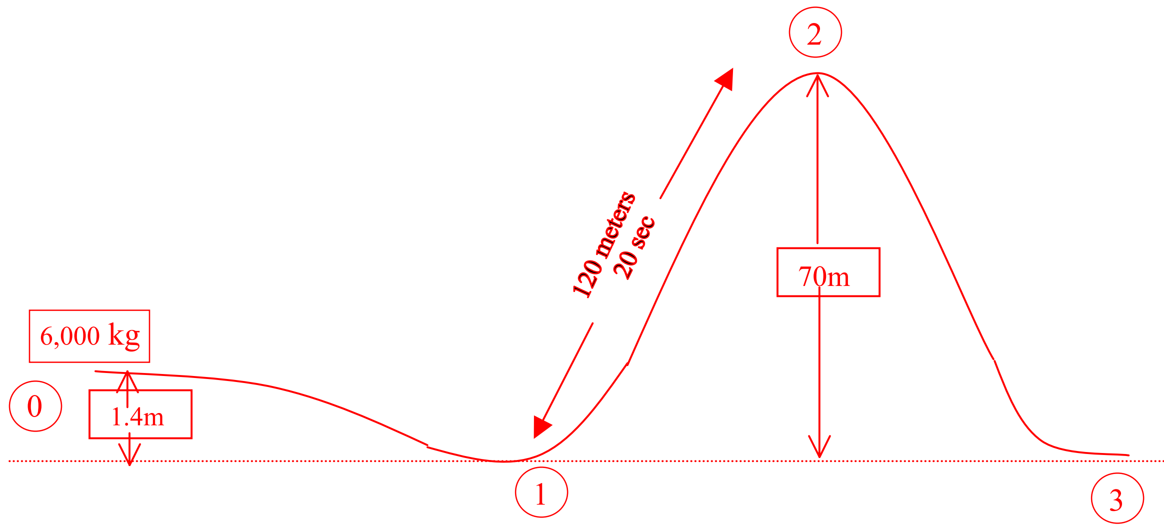
Roller Coaster Physics
Assignment for Lesson 6

When you board the ride, the coaster car is stopped and locked. The total mass of the car and passengers is 6,000 kg. The track initially slopes downward slightly. When the ride starts, the brakes are released and the car starts rolling down hill. At the bottom of the starting ramp (the car has dropped down 1.4 meters to ground level) and just as the train starts up the incline towards the top of the first hill, a moving chain grabs the bottom of the car. The lift pulls it up the 120-meter incline to the top of the first hill in 20 seconds. The first hill is 70 meters above ground level. The lifting mechanism releases the car at the top of the first hill and the car plunges downward!

<u>Answer the following</u>:

1. Draw a diagram and label all points of interest. Label the starting position as point 0, the bottom of the starting ramp/point of hook-up to the lifting mechanism as point 1, the top of the first hill/point where the lifting mechanism releases the car as point 2, and the bottom of the first hill as point 3.
2. How fast was the car going when it hooked onto the lifting mechanism?
3. How fast was the car going when the chain released at the top of the first hill?
4. How much work is needed to lift the car to the top of the first hill?
5. How much power is needed to lift the car to the top of the first hill?
6. How fast will the car be going at the bottom of the first hill?
7. What happens as the train hooks up to the chain lift? Assume the train travels 1.5 meters (rising up 1 meter) while it is changing speed to match that of the lift. Calculate the force on the coaster car.
8. Calculate the number of "g's" felt when they hook onto the chain. Explain what happens to the people and why.

1. Draw a diagram and label all points of interest. Label the starting position as point 0, the bottom of the starting ramp/point of hook-up to the lifting mechanism as point 1, the top of the first hill/point where the lifting mechanism releases the car as point 2, and the bottom of the first hill as point 3..



2. How fast was the car going when it hooked onto the lifting mechanism? Find v_1:

$$\cancel{KE_0}^{0} + PE_0 + \cancel{W_{0-1}}^{0} = KE_1 + \cancel{PE_1}^{0}$$

$$\cancel{m} g h_0 = \tfrac{1}{2} \cancel{m} v_1^2$$

$$v_1^2 = 2 g h_0$$

$$v_1 = \sqrt{2 g h_0} = \sqrt{(2)\left(9.8 \tfrac{m}{\sec^2}\right)(1.4 m)}$$

$$\boxed{v_1 = 5.24 \tfrac{m}{\sec}}$$

3. How fast was the car going when the chain released at the top of the first hill? Find v_2:

$$v_2 = \frac{d_{1-2}}{t}$$

$$= \frac{120 m}{20 \sec}$$

$$\boxed{v_2 = 6 \tfrac{m}{\sec}}$$

4. How much work is needed to lift the car to the top of the first hill? Find W_{1-2}:

$$KE_1 + \cancelto{0}{PE_1} + W_{1-2} = KE_2 + PE_2$$

$$\tfrac{1}{2}mv_1^2 + W_{1-2} = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$W_{1-2} = \tfrac{1}{2}mv_2^2 + mgh_2 - \tfrac{1}{2}mv_1^2$$

$$= \tfrac{1}{2}m\left[v_2^2 + 2gh_2 - v_1^2\right]$$

$$= (0.5)(6000m)\left[\left(6\tfrac{m}{\sec}\right)^2 + (2)\left(9.8\tfrac{m}{\sec^2}\right)(70m) - \left(5.24\tfrac{m}{\sec}\right)^2\right]$$

$$W_{1-2} = 4{,}141{,}627\,\mathrm{N}\cdot m \ \text{ or joules}$$

5. How much power is needed to lift the car to the top of the first hill?

Find P_{lift} :

$$P_{lift} = \frac{W_{1-2}}{t_{1-2}}$$

$$= \frac{4{,}141{,}627\,j}{20\sec}$$

$$= 207{,}081\ watts = 207.1\ kw$$

$$= 207.1\ \cancel{kw} \cdot \left(\frac{1.34\ hp}{1\ \cancel{kw}}\right)$$

$$\boxed{P_{lift} = 277.5\ hp}$$

6. How fast will the car be going at the bottom of the first hill? Find v_3:

$$KE_2 + PE_2 + \cancel{W_{2-3}}^{\,0} = KE_3 + \cancel{PE_3}^{\,0}$$

$$\tfrac{1}{2}\cancel{m}v_2^{\,2} + \cancel{m}gh_2 = \tfrac{1}{2}\cancel{m}v_3^{\,2}$$

$$v_3^{\,2} = v_2^{\,2} + 2gh_2$$

$$v_3 = \sqrt{v_2^{\,2} + 2gh_2}$$

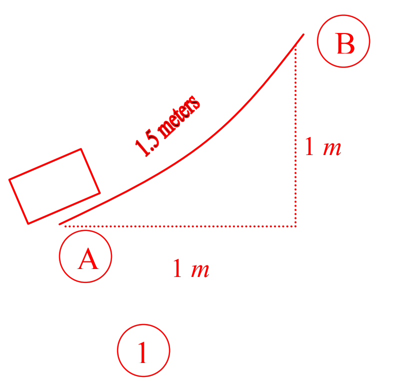
$$= \sqrt{\left(6\tfrac{m}{\sec}\right)^2 + (2)\cdot\left(9.8\tfrac{m}{\sec^2}\right)\cdot(70m)}$$

$$v_3 = 37.5\tfrac{m}{\sec}$$

$$= \left(37.5\tfrac{\cancel{m}}{\cancel{\sec}}\right)\cdot\left(\frac{2.2\tfrac{mi}{hr}}{1\tfrac{\cancel{m}}{\cancel{\sec}}}\right)$$

$$\boxed{v_3 = 82.5\tfrac{mi}{hr}}$$

7. What happens as the train hooks up to the chain lift? Assume the train travels 1.5 meters (rising up 1 meter) while it is changing speed to match that of the lift. Calculate the force on the coaster car.



$$KE_A + \cancel{PE_A}^{\,0} + W_{A-B} = KE_B + PE_B$$

$$\tfrac{1}{2}mv_A^{\,2} + W_{A-B} = \tfrac{1}{2}mv_B^{\,2} + mgh_B$$

$$W_{A-B} = \tfrac{1}{2}mv_B^{\,2} - \tfrac{1}{2}mv_A^{\,2} + mgh_B$$

$$= \tfrac{1}{2}m\left[v_B^{\,2} - v_A^{\,2} + 2gh_B\right]$$

$$= \tfrac{1}{2}(6000kg)\left[\left(6\tfrac{m}{\sec}\right)^2 - \left(5.24\tfrac{m}{\sec}\right)^2 + (2)\left(9.8\tfrac{m}{\sec^2}\right)(1m)\right]$$

$$W_{A-B} = 84{,}427\, N\cdot m \text{ or } joules$$

But, we know the relation between work and force, so:

$$W_{A-B} = F\cdot d_{A-B}$$

$$F = \frac{W_{A-B}}{d_{A-B}} = \frac{84{,}427\ N\cdot m}{1.5m}$$

$$\boxed{F = 56{,}285\, N}$$

8. Calculate the number of "g's" felt when they hook onto the chain. Explain what happens to the people and why.

a. We can determine the acceleration of the coaster:

$$F = m \cdot a$$

$$a = \frac{F}{m} = \frac{56,285\,N}{6,000kg} = \frac{56,285\,\frac{\cancel{kg}\cdot m}{\sec^2}}{6,000\,\cancel{kg}} = 9.4\,\frac{m}{\sec^2}$$

Finally, we can determine how many g's is the car being subjected to:

$$\#g's\ \text{felt} = \frac{\text{acceleration felt}}{\text{acceleration of gravity}} = \frac{9.4\,\frac{m}{\sec^2}}{9.8\,\frac{m}{\sec^2}}$$

$$\boxed{\#g's\ \text{felt} = 0.96g's}$$

b. The 56,285 *N* force found in part 7. above causes the car's speed to increase from $5.24\,\frac{m}{\sec}$ to $6\,\frac{m}{\sec}$ in order to match the speed of the chain lift. As a result, the roller coaster car is accelerated forward but, because of Newton's first law - which says that a person's velocity will remain unchanged until acted on by an outside force - the passengers remain at the slower speed and are therefore pushed back into their seats at an acceleration of $9.4\,\frac{m}{\sec^2}$ (feeling 0.96 g's) and are stopped by the back of the their seat.