Lesson 3
Class Notes
Introduction to Roller Coasters

We know that:

$KE_1 = \frac{1}{2} m v_1^2$ $PE_1 = mgh_1$

$KE_2 = \frac{1}{2} m v_2^2$ $PE_2 = mgh_2$

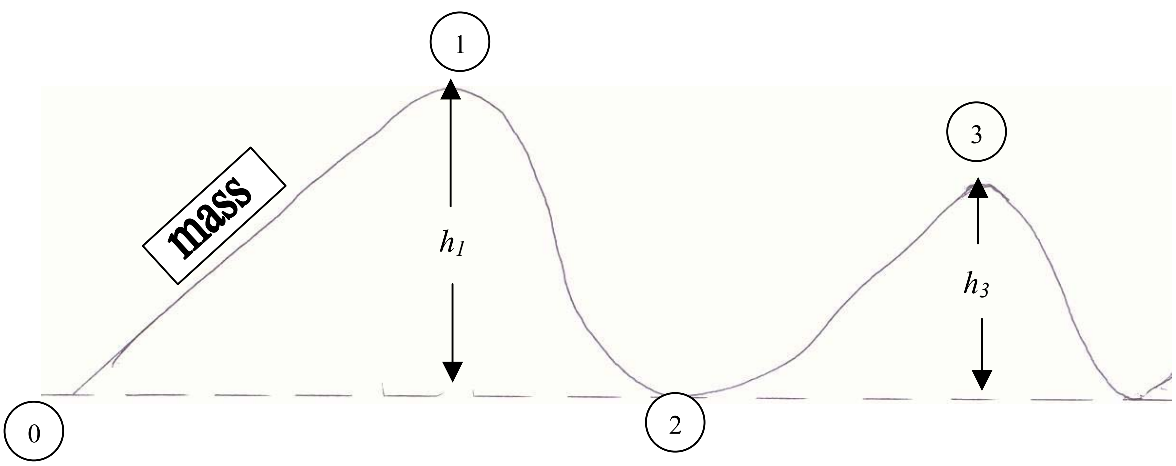
Where:

$KE_1 \equiv$ Kenetic Energy at pt 1 $m \equiv$ mass (kg) or (lb)

$KE_2 \equiv$ Kenetic Energy at pt 2 $v_1 \equiv$ velocity at pt 1 ($\frac{m}{sec}$) or ($\frac{ft}{sec}$)

$PE_1 \equiv$ Potential Energy at pt 1 $h_2 \equiv$ height of pt 2 above the reference level (m) or (ft)

$PE_2 \equiv$ Potential Energy at pt 2 $g \equiv$ acceleration due to gravity ($9.8 \frac{m}{sec^2}$) or ($32.2 \frac{ft}{sec^2}$)



Now, when we neglect friction and windage, $KE_1 + PE_1 = KE_2 + PE_2$ because total energy can not be created or destroyed. That is, the total energy at any point (say at point 1) will be the same as the total energy at any other point (say at point 2). Similarly, for example, we can also say that $KE_2 + PE_2 = KE_3 + PE_3$, or $KE_1 + PE_1 = KE_3 + PE_3$.

Solve some examples.

1. Given: $m = 6,000kg$, $v_1 = 0\frac{m}{\sec}$, $h_1 = 65m$, and $h_2 = 0m$

   Find: v_2

   Work:

   Method 1 – "Non-Algebraic" Approach.

$$KE_1 + PE_1 = KE_2 + PE_2$$

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$\tfrac{1}{2}(6000kg)(0\tfrac{m}{\sec})^2 + (6000kg)(9.8\tfrac{m}{\sec^2})(65m) = \tfrac{1}{2}(6000kg)v_2^2 + (6000kg)(9.8\tfrac{m}{\sec^2})(0m)$$

$$3822000\frac{kg \cdot m^2}{\sec^2} = 3000v_2^2 kg$$

$$\frac{3822000\frac{kg \cdot m^2}{\sec^2}}{3000kg} = v_2^2$$

$$1274\frac{m^2}{\sec^2} = v_2^2$$

$$35.69\frac{m}{\sec} = v_2$$

Method 2 – "Algebraic" Approach.

$$KE_1 + PE_1 = KE_2 + PE_2$$

$$\tfrac{1}{2}\cancel{m}v_1^{\ 2} + \cancel{m}gh_1 = \tfrac{1}{2}\cancel{m}v_2^{\ 2} + \cancel{m}gh_2$$

$$\tfrac{1}{2}\cancel{v_1^{\ 2}}^{\,0} + gh_1 = \tfrac{1}{2}v_2^{\ 2} + g\,\cancel{h_2}^{\,0}$$

$$gh_1 = \tfrac{1}{2}v_2^{\ 2}$$

$$2gh_1 = v_2^{\ 2}$$

$$v_2^{\ 2} = 2gh_1$$

$$v_2^{\ 2} = 2\cdot(9.8\tfrac{m}{\sec^2})\cdot(65m) = 1274\tfrac{m^2}{\sec^2}$$

$$v_2 = \sqrt{1274\tfrac{m^2}{\sec^2}} = \left(\sqrt{1274}\right)\cdot\left(\sqrt{\tfrac{m^2}{\sec^2}}\right)$$

$$v_2 = 35.69\tfrac{m}{\sec}$$

2. Given: $m = 4{,}500kg$, $v_1 = 3\frac{m}{\sec}$, $v_2 = 75\frac{m}{\sec}$, and $h_2 = 0m$

Find: h_1

Work: A "Hybrid" Approach.

Steps:

1. Write General Energy Balance Equation between two points on the coaster.
2. Substitute in the definitions of KE, PE, W, etc.
3. Look for common terms (usually mass) to divide out.
4. Look for terms equal to zero (velocity, height, work, etc.)
5. Write simplified equation and solve.

1. $KE_1 + PE_1 = KE_2 + PE_2$

2. $\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

3. $\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gh_2$

4. $\frac{1}{2}v_1^2 + gh_1 = \frac{1}{2}v_2^2 + g\cancel{h_2}^{\,0}$

5. $\frac{1}{2}v_1^2 + gh_1 = \frac{1}{2}v_2^2$

Continue with a Non-Algebraic Approach:

$$\frac{1}{2}v_1^2 + gh_1 = \frac{1}{2}v_2^2$$

$$\frac{1}{2}\left(3\frac{m}{\sec}\right)^2 + \left(9.8\frac{m}{\sec^2}\right)h_1 = \frac{1}{2}\left(75\frac{m}{\sec}\right)^2$$

$$4.5\left(\frac{m^2}{\sec^2}\right) + \left(9.8\frac{m}{\sec^2}\right)h_1 = 2812.5\left(\frac{m^2}{\sec^2}\right)$$

$$\left(9.8\frac{m}{\sec^2}\right)h_1 = 2808\left(\frac{m^2}{\sec^2}\right)$$

$$h_1 = \frac{2808\frac{m^2}{\sec^2}}{9.8\frac{m}{\sec^2}}$$

$$h_1 = 286.53m$$

Continue with an Algebraic Approach:

$$\tfrac{1}{2}{v_1}^2 + gh_1 = \tfrac{1}{2}{v_2}^2$$

$$\tfrac{1}{2}\left(3\tfrac{m}{\sec}\right)^2 + \left(9.8\tfrac{m}{\sec^2}\right)h_1 = \tfrac{1}{2}\left(75\tfrac{m}{\sec}\right)^2$$

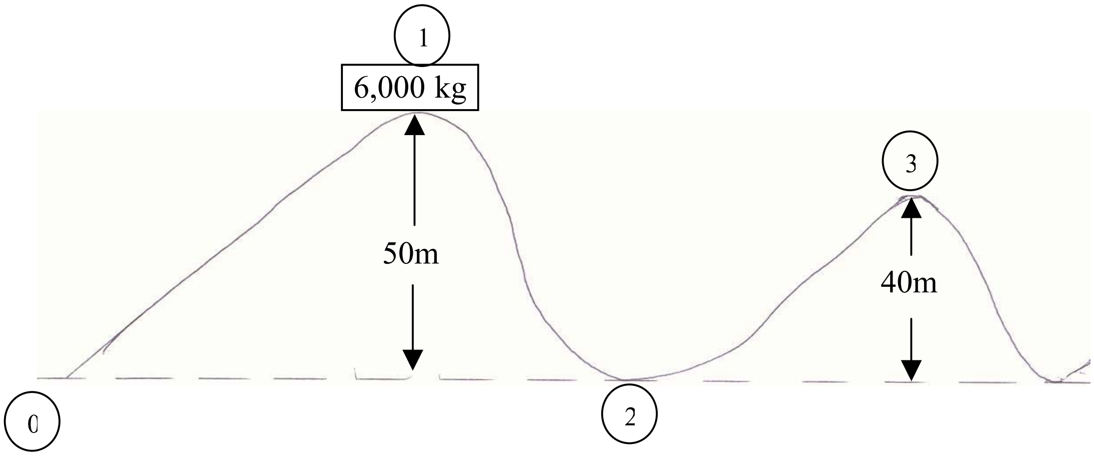
$$4.5\left(\tfrac{m^2}{\sec^2}\right) + \left(9.8\tfrac{m}{\sec^2}\right)h_1 = 2812.5\left(\tfrac{m^2}{\sec^2}\right)$$

$$\left(9.8\tfrac{m}{\sec^2}\right)h_1 = 2808\left(\tfrac{m^2}{\sec^2}\right)$$

$$h_1 = \frac{2808\tfrac{m^2}{\sec^2}}{9.8\tfrac{m}{\sec^2}}$$

$$h_1 = 286.53m$$

3. A roller coaster train and passengers has a mass of 6,000 kg and is held in place at the top of the first hill. This hill is 50 meters above the lowest point on the track. When the coaster is released, it speeds down a 65-degree slope to the bottom of the first hill, which is the lowest point on the ride. How fast will the train be going when it reaches the bottom of the hill? (Neglect friction)



1. $KE_1 + PE_1 = KE_2 + PE_2$

(note: the 65° slope has no effect on the problem.)

2. $\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

3. $\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gh_2$

4. $\frac{1}{2}\cancelto{0}{v_1^2} + gh_1 = \frac{1}{2}v_2^2 + g\cancelto{0}{h_2}$

5. $gh_1 = \frac{1}{2}v_2^2$

$$v_2^2 = 2gh_1 = 2\cdot\left(9.8\tfrac{m}{\sec^2}\right)\cdot(50m) = 980\tfrac{m^2}{\sec^2}$$

$$v_2 = \sqrt{980\tfrac{m^2}{\sec^2}} = \left(\sqrt{980}\right)\cdot\left(\sqrt{\tfrac{m^2}{\sec^2}}\right)$$

$$v_2 = 31.3\tfrac{m}{\sec}$$